SELF-SIMILAR ELECTRICAL SKIN EXPLOSION OF A CONDUCTOR

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We examine the problem of the electrical explosion of a conductor with flat boundary in a strong magnetic field. We estimate the role of heat conduction in order to determine the critical electrical fields in which fusion and vaporization of the metal take place. The characteristic features of the explosion of a layered medium are examined.

The physical picture of conductor explosion under the action of a high-density current is described in [1, 2]. Calculations are made in [2, 3] under the assumption that the current is distributed uniformly across the section of the conductor. In the case of fast processes, in which the current rise time is less than the effective penetration time of the magnetic field into the conductor ("skin" time), the thin surface layer explodes and the vaporization boundary travels into the conductor. In this case the use of characteristics averaged across the conductor section no longer reflects satisfactorily the essence of the phenomenon. The assumption that the conductivity-loss front (vaporization boundary) travels in the conductor with the speed of sound [2] also cannot always be made. In fact, the rate of growth of the skin layer, which determines the velocity of the thermal wave into the depth of the conductor, is proportional to $1/\sqrt{t}$, i.e., it sooner or later becomes less than the speed of sound. From this instant on, the vaporization-front velocity will be determined by the magnetic field diffusion processes and not by the elastic properties of the conductor.

In the following we examine self-similar propagation of the vaporization wave (more precisely, the conductivity-loss wave, or the electron-decollectivization wave following the terminology of [2]). It is assumed that heat conduction and motion of the medium are negligible and that the electrical conductivity is independent of the temperature up to the vaporization point, upon reaching which the conductivity disappears.

1. Self-similar Problem of Conductor Explosion. Let us assume that in the initial state the conductor fills the half-space $0 \le x < \infty$. The magnetic field at the boundary of the conductor is parallel to its surface and equal to $H_0(t)$. At the initial time the field in the conductor is zero. The conductivity σ remains constant until vaporization at time t_0, at which a unit volume of the conductor obtains, as a result of current heating, the heat of sublimation Q_0, after which $\sigma = 0$. In this process a vaporization wave, whose equation of motion $x = X(t)$ is unknown, propagates into the depth of the conductor. Under these assumptions and neglecting displacement currents, determination of the magnetic field intensity H in the conductor reduces to the solution of the diffusion equation

$$\frac{\partial H}{\partial t} = \frac{c^2}{4\pi\sigma}\frac{\partial^2 H}{\partial x^2} \tag{1.1}$$

in the region $t \ge 0$, $X(t) \le x < \infty$ (Fig. 1) satisfying the conditions

$$H(x, 0) = 0, \quad H(x, t)_{x=X(t)} = H_0(t), \qquad Q(x, t)_{x=X(t)} = Q_0 \tag{1.2}$$

Here Q(x, t) is the heat released per unit volume of the conductor in the process of its heating by the current. Generally speaking, the exact solution of the problem of heating of a conductor by a skin current reduces to the solution of the heat-conduction equation with Joule heat sources of intensity

$$\frac{J^2}{\sigma} = \frac{c^2}{16\pi^2\sigma}\left(\frac{\partial H}{\partial x}\right)^2 \tag{1.3}$$

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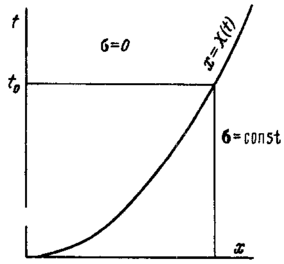

Fig. 1

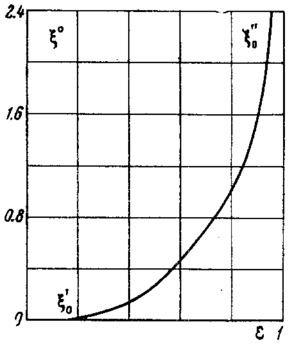

Fig. 2

However, it was shown in [4] that for metallic conductors we can neglect heat conduction; this makes it possible to rewrite the last vaporization condition (1.2) in the form

$$Q_0 = \frac{c^2}{16\pi^2\sigma} \int_0^{t_0} \left(\frac{\partial H}{\partial x}\right)^2 dt \tag{1.4}$$

The formulated problem has a self-similar solution for constant H_0 and Q_0. In this case we can form the following dimensionless combinations from the defining parameters of the problem and the variables x and t:

$$\xi = \frac{\sqrt{\pi\sigma}\, x}{c\sqrt{t}}, \quad h = \frac{H}{H_0}, \quad \varepsilon = \frac{H_0^2}{8\pi Q_0} \tag{1.5}$$

This implies that the solution depends only on the single variable ξ. Then (1.1) reduces to

$$h'' = -2\xi h' \tag{1.6}$$

and the initial condition (1.2) yields

$$h(\infty) = 0 \tag{1.7}$$

while the boundary condition (1.2) reduces to

$$h(\xi)_{x = X(t)} = 1 \tag{1.8}$$

On the vaporization wave front, ξ equals some unknown constant ξ_0, and the equation of motion for the vaporization front is

$$X(t) = \xi_0 \frac{c}{\sqrt{\pi\sigma}} \sqrt{t} \tag{1.9}$$

The solution of (1.6) with conditions (1.7) and (1.8) is

$$h(\xi) = \frac{1 - \Phi(\xi)}{1 - \Phi(\xi_0)} \tag{1.10}$$

where $\Phi(\xi)$ is the probability integral.

The vaporization condition (1.4) makes it possible to find ξ_0 as a function of the parameter ε and thereby calculate the conductor decomposition rate. Substitution of (1.10) into (1.4) leads in our self-similar problem to the equation

$$\varepsilon = -\frac{\pi}{2} \frac{[1 - \Phi(\xi_0)]^2}{Ei(-2\xi_0^2)} \tag{1.11}$$

where Ei(z) is the integral exponential function. The curve of ξ_0 versus ε is shown in Fig. 2, where

$$\xi_0' = \frac{1}{\sqrt{2\beta}} \exp\left(-\frac{\pi}{4\varepsilon}\right), \quad \beta = 1.781 \quad \text{for } \varepsilon \ll 1 \tag{1.12}$$

$$\xi_0'' = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{1-\varepsilon}} \quad \text{for } \varepsilon \sim 1 \tag{1.13}$$

2. Minimum Field Causing Vaporization. Limits of Applicability of Self-similar Solution. The resulting solution formally yields the explosion of the skin layer for any small small magnetic fields H_0, although it is physically obvious that this cannot take place. This situation occurred because of the neglect of heat conduction and representation of the vaporization condition in the simplified form (1.4). This is easily shown as follows. As is known [5], the skin layer thickness δ and the current density j at the surface of the conductor is, respectively,

$$\delta \sim \frac{c\sqrt{t}}{\sqrt{\pi\sigma}}, \qquad j \sim \frac{c}{4\pi} \frac{H_0}{\delta} \sim \frac{H_0}{\sqrt{4\pi}} \left(\frac{\sigma}{t}\right)^{1/2}$$

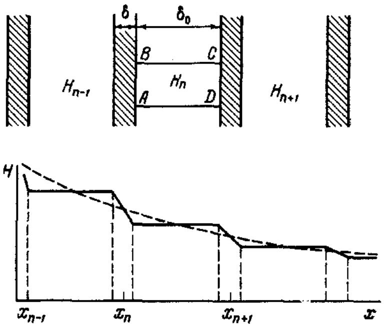

Fig. 3

Neglecting heat conduction, we can easily show that in unit volume at the conductor surface during the time from t_1 to t_2 there is released the heat

$$Q = \int_{t_1}^{t_2} \frac{j^2}{\sigma} dt \sim \frac{H_0^2}{4\pi} \ln \frac{t_2}{t_1} \tag{2.1}$$

and even small currents as $t_2 \to \infty$ lead to sufficient heating and vaporization of the conductor. However, heating over a long period, even for low thermal conductivity, can in no way be assumed adiabatic. Accounting for heat conduction makes it possible to find the critical field H* which is sufficient for initiation of an explosion and thereby establishes the limits of applicability of the resulting solution.

The temperature variation, like the variation of the magnetic field in the conductor, is described by the diffusion Eq. (1.1) if we replace the field diffusion coefficient $D_m = c^2/4\pi\sigma$ by the thermal diffusivity coefficient $D_T = \varkappa/q$, where $\varkappa$ is the thermal conductivity and q the heat capacity per unit volume of the conductor. According to (1.9), the thickness of the layer evaporated by the time t is

$$\delta = \xi_0 \frac{c}{\sqrt{\pi\sigma}} \sqrt{t} \tag{2.2}$$

The characteristic time τ for establishing thermal equilibrium of such a layer with the ambient medium is

$$\tau \sim \frac{\delta^2}{D_T} = \frac{4\xi_0^2 D_m}{D_T} t \tag{2.3}$$

The condition of applicability of the adiabatic approximation in examining skin-layer vaporization for $\tau > t$ leads to $\xi_0^2 > D_T/4D_m$; hence for weak fields, after substitution of (1.12) it is not difficult to obtain

$$H^* > 2\pi \left(\frac{Q_0}{\ln (2D_m / \beta D_T)} \right)^{1/2} \tag{2.4}$$

Using the connection between thermal conductivity and electrical conductivity in accordance with the Wiedemann-Franz law and, naturally, assuming the metal temperature equal to the boiling temperature T_{**}, we can rewrite the resulting estimate of the minimum field necessary to initiate the vaporization process in the form

$$H^* > 2\pi \sqrt{Q_0} \left(\ln \frac{\pi}{6\beta} \left(\frac{ck}{e} \right)^2 \frac{qT_{**}}{\varkappa^2} \right)^{-1/2} \tag{2.5}$$

Here e is the electron charge and k is Boltzmann's constant. Calculation of the critical fields by means of (2.4) yield for copper $H^* > 1.5 \cdot 10^6$ Oe, for tungsten $H^* > 1.9 \cdot 10^6$ Oe, and for lead $H^* > 0.66 \cdot 10^6$ Oe.

For magnetic fields $H \sim \sqrt{8\pi Q_0}$ the obtained solution leads to infinite velocity of fusion-wave propagation into the depth of the conductor. It is clear that in this case new physical processes associated with the dispersion velocity of the vaporizing metal begin to play a role and significantly alter the phenomenon. In fact, in the self-similar solution we assumed Q_0 = const, which is possible with constant pressure at the vaporization boundary. However, if the dispersion of the metal vapors takes place slowly in comparison with the velocity of the vaporization front, then the pressure at this front increases in the course of time, which leads to increase of the boiling temperature and increase of Q_0 (the effect observed in the experiments of Kvartskhava [6]). The increase of Q_0 for given H_0 reduces the parameter ε and leads to the establishment in accordance with this of some finite velocity of the vaporization front. Thus, the problem of conductor vaporization in very strong fields will be basically hydrodynamic and requires special examination.

3. Conductor Fusion. The self-similar problem of conductor melting without a change of conductivity can be examined analogously to the vaporization problem. In this case the boundary of the conductor does not change but the fusion front propagates into the depth of the conductor and corresponds to some constant value $\xi = \xi_1$ of the self-similar variable. The essential difference in the mathematical formulation of this problem from that examined above lies in replacement of condition (1.8) by $h(0) = 1$, which leads in (1.10) to the denominator becoming unity and a corresponding change in (1.11). In this case we find that

$$\varepsilon_1 = -\frac{\pi}{2}\frac{1}{Ei(-2\xi_1^2)} \tag{3.1}$$

The critical field H_1^* which causes melting of the conductor is defined as before by (2.5) with replacement of the heat of vaporization Q_0 by the heat of fusion Q_1 and replacement of the boiling temperature T_{**} by the melting temperature T_*. The magnitudes of the critical fields amount to $5.5 \cdot 10^5$ Oe for copper and $1.6 \cdot 10^5$ Oe for lead.

According to these calculations, pulsed solenoids made from copper can withstand fields up to $0.5 \cdot 10^6$ Oe without failure (except for possible mechanical deformations).

In $(0.5-1.5) \cdot 10^6$ Oe fields the copper surface must melt with possible formation of liquid jets. In fields higher than $1.5 \cdot 10^6$ Oe vaporization of the copper occurs, accompanied by pressure rise, formation of shock waves, and expansion of the vapors.

4. Explosion of Layered Material. It is of interest to examine magnetic field diffusion into a layered medium consisting of alternating layers of conductor and dielectric. Neglecting the high-frequency wave pictures, we can assume that the field is uniform in each layer of the dielectric, which for a definite magnitude of the current diffusion through some particular conductor layer leads to reduction of the field intensity at the inner surface of the conducting layer and, therefore, to increase of the current density in this layer. The result is an increase of the power heating the conductor and its vaporization occurs earlier. It is not difficult to obtain the equation describing field diffusion into the layered medium if we assume that the conducting layer thickness δ and the distances δ_0 between the layers are small. Let the field in the n-th dielectric layer be $H_n(t)$ and the dielectric constant equal unity. The electromagnetic induction law for the rectangular contour ABCD, whose boundaries AB and CD run along the inner surface of the n-th layer and along the outer surface of the (n + 1)-st layer of the conductor (Fig. 3), can be written in the form

$$\frac{1}{c}\delta_0\frac{\partial H_n}{\partial t} = \frac{c}{4\pi\sigma}\left(\frac{\partial H}{\partial x}\bigg|_{CD} - \frac{\partial H}{\partial x}\bigg|_{AB}\right) \tag{4.1}$$

For small δ we can find the value of the derivative $\partial H/\partial x$ on boundaries AB and CD of the conductors by expanding the field of the n-th and (n + 1)-st layers of the conductor into a Taylor series and retaining only the first three terms of this series. Thus, for finding $(\partial H/\partial x)_{AB}$ we have

$$H_{n-1} = H_n - \delta\frac{\partial H}{\partial x}\bigg|_{AB} + \frac{\delta^2}{2}\frac{\partial^2 H}{\partial x^2}\bigg|_{AB}$$

By virtue of the equation of field diffusion into the n-th conductor layer and the continuity of H on boundary AB

$$\frac{\partial^2 H}{\partial x^2}\bigg|_{AB} = \frac{4\pi\sigma}{c^2}\frac{\partial H_n}{\partial t}, \qquad \frac{\partial H}{\partial x}\bigg|_{AB} = \frac{H_n - H_{n-1}}{\delta} + \frac{\delta}{2}\frac{4\pi\sigma}{c^2}\frac{\partial H_n}{\partial t}$$

Similar calculations yield

$$\frac{\partial H}{\partial x}\bigg|_{CD} = \frac{H_{n+1} - H_n}{\delta} - \frac{\delta}{2}\frac{4\pi\sigma}{c^2}\frac{\partial H_n}{\partial t}$$

and substitution of the resulting values of $\partial H/\partial x$ on the boundaries into (4.1) leads to

$$(\delta_0 + \delta)\frac{\partial H_n}{\partial t} = \frac{c^2}{4\pi\sigma}\frac{1}{\delta}(H_{n+1} + H_{n-1} - 2H_n) \tag{4.2}$$

Replacing the stepwise distribution of the field in the dielectric by continuous distribution H(x, t) (shown dashed in Fig. 3), taking at the center of the n-th dielectric layer the value $H_n(t)$, and assuming $(\delta_0 + \delta)$ small, we can rewrite (4.2) in the form

$$\frac{\partial H}{\partial t} = \frac{c^2}{4\pi\sigma}\frac{\delta_0 + \delta}{\delta}\frac{\partial^2 H}{\partial x^2} \tag{4.3}$$

i.e., field diffusion into the layered medium in first approximation corresponds to field diffusion into a continuous conductor with conductivity

$$\sigma_c = \alpha\sigma, \quad \alpha = \frac{\delta}{\delta_0 + \delta}$$

Here α is the conductor fraction in the layered medium. To the same accuracy the current density in the n-th conductor layer is

$$J_n = \frac{c}{4\pi}\frac{H_n - H_{n-1}}{\delta} = \frac{c}{4\pi}\frac{\delta_0 + \delta}{\delta}\frac{\partial H}{\partial x}\bigg|_{x=x_n} \tag{4.4}$$

and the vaporization condition

$$Q_0 = \int_0^{t_0} \frac{J^2}{\sigma}\,dt = \frac{1}{\alpha}\frac{c^2}{16\pi^2\sigma_c}\int_0^t \left(\frac{\partial H}{\partial x}\right)^2 d. \tag{4.5}$$

corresponds to (1.4) for vaporization of a solid conductor with conductivity σ_c and $Q_c = \alpha Q_0$.

As in section 1, we can pose the self-similar problem of explosion of a layered medium. Its solution is described by (1.9) – (1.11) with σ, ε, ξ_0 replaced by the effective characteristics of the layered medium, defined by the relations

$$\sigma_c = \alpha\sigma, \qquad \varepsilon_c = \frac{\varepsilon}{\alpha}, \qquad \xi_{0c} = \xi_0\left(\frac{\varepsilon}{\alpha}\right) \tag{4.6}$$

The ratio of the vaporized conductor mass M_c in the layered medium case to the mass M of the continuous conductor vaporized in the same time for a given field intensity at the boundary of the medium is

$$\frac{M_c}{M} = \frac{\alpha X_c(t)}{X(t)} = \alpha\left(\frac{\sigma}{\sigma_c}\right)^{1/2}\frac{\xi_{0c}}{\xi_0} = \sqrt{\alpha}\,\frac{\xi_0(\varepsilon/\alpha)}{\xi_0(\varepsilon)} \tag{4.7}$$

For small ε and ε/α by virtue of (1.12)

$$\frac{M_c}{M} = \sqrt{\alpha}\exp\frac{\pi(1-\alpha)}{4\varepsilon} \tag{4.8}$$

For $\alpha = 1/3$, $\varepsilon = 1/5$, as an example, we obtain $M_c/M \approx 8$.

The times for destruction of a definite conductor mass for a given external field are connected by the relation

$$\frac{\tau}{\tau_c} = \alpha\frac{\xi_0^2(\varepsilon/\alpha)}{\xi_0^2(\varepsilon)} \tag{4.9}$$

For small ε and ε/α

$$\frac{\tau}{\tau_c} = \alpha\exp\frac{\pi(1-\alpha)}{2\varepsilon} \tag{4.10}$$

For $\alpha = 1/3$ and $\varepsilon = 1/5$ this ratio is equal to about 64, i.e., explosion of the layered conductor takes place tens of times faster.

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